Due Date: 22.11.2019 – Monday – Morning 10:00 AM

HOMEWORK 4

Que1: (10 p) Assume that $x, y \in R$ and $z = x + iy \in C$ and answer the followings:

Evaluate each of these integrals, where the path is an arbitrary contour between the limits of integration:

(a) $\int_{i}^{i/2} e^{\pi z}\, dz$; (b) $\int_{0}^{\pi+2i} \cos\left(\frac{z}{2}\right) dz$; (c) $\int_{1}^{3} (z-2)^3\, dz$.

Que2:(20p)

For the contours C and functions f in Exercises 1 to 6, use parametric representations for C, or legs of C, to evaluate

$$\int_C f(z)\, dz\,.$$

1. $f(z) = (z+2)/z$ and C is
   (a) the semicircle $z = 2e^{i\theta}$ $(0 \leqq \theta \leqq \pi)$;
   (b) the semicircle $z = 2e^{i\theta}$ $(\pi \leqq \theta \leqq 2\pi)$;
   (c) the circle $z = 2e^{i\theta}$ $(0 \leqq \theta \leqq 2\pi)$.

Que3: (30 p) Assume that $x, y \in R$ and $z = x + iy \in C$ and answer the followings:

1. Let C denote the boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$, where C is described in the positive sense. Evaluate each of these integrals:

   (a) $\int_C \frac{e^{-z}\, dz}{z - (\pi i/2)}$; (b) $\int_C \frac{\cos z}{z(z^2+8)}\, dz$; (c) $\int_C \frac{z\, dz}{2z+1}$;

Que4: (20 p) : C is the arc from 1+i to 3+9i along the curve $y = x^2$ and

$$f(z) = \begin{cases} 2x, & 0 < x < 2 \\ y, & 2 < x < 3 \end{cases}$$

Calculate the $\oint f(z)dz$ on counter C.

Que5: (20 p):

11. Evaluate $\oint_C (\bar{z} - 3)\, dz$ where C is the indicated closed curve along the first quadrant part of the circle $|z| = 2$, and the indicated parts of the x and y axes. *Hint:* Don't try to use Cauchy's theorem! (Why not? *Further hint:* See Problem 2.3.)